GRAVITATIONAL STIRRING OF PLANETESIMALS BY A PLANET EMBRYO: IMPLICATIONS FOR RUNAWAY GROWTH

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In a recent work on planetary accretion [1] it was suggested that the largest body in a distribution of embryos would be an ineffective perturber of planetesimals due to the conservation of the Jacobian integral and effective stirring of planetesimals would pass to the next largest body. We wish to determine under what conditions this would be true and apply the results to accretion in the asteroid belt in the presence of an already formed Jupiter embryo.

For convenience of calculation we start with an "asteroid" of mass $m=2\times10^{28}$ g at 2.7 AU with a Hill sphere radius of $r_H=0.04$ AU. We integrate to find the trajectory of a nearby planetesimal displaced by $(\Delta a/a)=2/3$ with eccentricity $e=1$ and inclination $i=1/2$ (in units of r_H) and determine the square of the random velocity v_r given by

$$v_r^2 = (5e^2/8) + (i^2/2) \tag{1}$$

(velocities are divided by the keplerian velocity). The changes from the initial value $v_r^2=0.75$ after each encounter with the embryo for ten phase averaged trajectories is shown by the jagged line in Fig. 1. We may compare this result with the expected velocity resulting from a 2–D random walk in (e,i) space where the v_r^2 displacement per encounter is simply

$$\delta v_r^2 \simeq 4g^2m^2/v^2d^2 \tag{2}$$

shown by the smooth curve in Fig. 1. We determine v^2 and d^2 analytically by phase averaging where v is the relative encounter velocity including keplerian shear and d is the impact parameter. Assuming $i=e/2$ we have

$$v^2d^2 = [v_r^2+\tfrac{9}{4}(\Delta a/a)^2][(\Delta a/a)^2+\tfrac{5}{6}v_r^2], \text{ with } (\Delta a/a)=2/3. \tag{3}$$

We see that v_r^2 in our phase averaged trajectories are less than those expected from a random walk and appear to reach a constant value of $v_r^2-0.75 \approx 1$.

The velocity displacements are not random because of conservation of the Jacobian integral in the restricted 3–body problem given by

$$J \approx \tfrac{3}{8}(\Delta a/a)^2 - \tfrac{1}{2}e^2 - \tfrac{1}{2}i^2. \tag{4}$$

Indeed, we find the correlation coefficient between displacements δe^2 and δi^2 for the trajectories in Fig. 1 is 0.999 when $|\delta(\Delta a/a)^2| < |\delta e^2+\delta i^2|/4$. Under this condition $\delta J \approx -\frac{1}{2}(\delta e^2+\delta i^2) = 0$ resulting in the correlation $\delta e^2 \approx -\delta i^2$. Because of this correlation, we actually have a 1–D random walk in (e,i) space which can be expected to approach a constant v_r^2 after many encounters.

We can eliminate this correlation if we have perturbations by a fourth body of mass m′ also giving rise to displacements δv_r^2 in Eq. (2) so that $\Delta J \approx J$ in Eq.(4). We find that

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$\delta v_r^2 \approx \delta v_r'^2$ when

$$m'/v'd' \approx m/vd. \quad (5)$$

For the case at hand, this condition corresponds to a "Jupiter" embryo at 5.2 AU with mass $m'=8\times10^{30}$g. The random planetesimal velocities for this 4–body problem with 20 phase averaged trajectories in Fig. 2 show good agreement with the expected 2–D random walk results. Indeed, we find the correlation coefficient between displacements δe^2 and δi^2 under the above conditions is now only 0.038 and $\Delta J_{rms} > J_{average}$.

We may readily use Eq. (5) to scale m and m′ to realistic values for the present asteroid belt (note that m/vd scales like $m^{1/3}$ in r_H units). We find that a Ceres size body will be an effective stirrer in the presence of a Jupiter embryo larger than 50 earth masses ($m' \geq 3\times10^{29}$ g). This means in the presence of Jupiter even the largest asteroid embryo in any accretion zone can effectively stir planetesimals. It has been shown [2,3] that such stirring by an accreting embryo can result in large planetesimal velocities and consequent self–termination of the embryo's runaway accretion. Large planetesimal velocities can lead to significant mass loss due to collisional disruption and subsequent orbital decay due to gas drag; this helps to explain the present mass distribution in the asteroid belt. Furthermore, runaway growth in the outer planet region can be hindered by the presence of Jupiter making resonance accretion [4] a more likely outcome.

[1] G. W. Wetherill and G. R. Stewart (1989). Accumulation of a swarm of small planetesimals. Icarus (in press). [2] C. Patterson and D. Spaute (1988). Self–termination of runaway growth in the asteroid belt. Asteroids II Conference Proceedings. [3] C. Patterson and D. Spaute (1988). Planetary accretion by runaway growth. Origin of the Earth Conference Proceedings. [4] C. Patterson (1987). Resonance capture and the evolution of the planets. Icarus 70, 319–333.